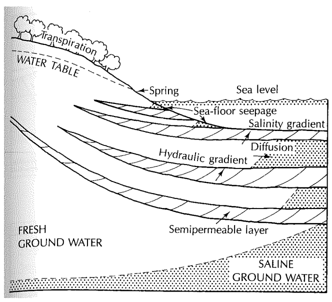
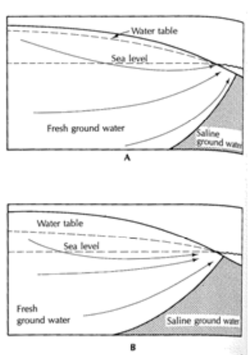
Coastal Aquifers



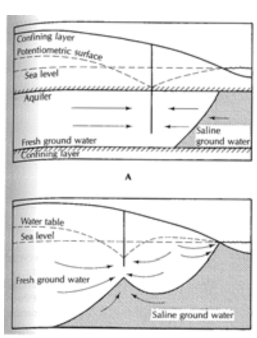
◄ FIGURE 8.33
Typical fresh-water–salt-water relationship in a layered coastal aquifer.

Passive Salt Water Intrusion

► FIGURE 8.34
A. Unconfined coastal aquifer under natural ground-water discharge conditions. **B.** Passive saline-water encroachment due to a general lowering of the water table. Flow in the fresh-water zone is still seaward.



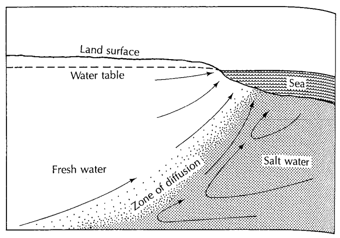
Active Salt Water Intrusion



◄ FIGURE 8.35
A. Active saline-water encroachment in a confined aquifer with the potentiometric surface below sea level. **B.** Active saline-water encroachment in an unconfined aquifer with the water table drawn below sea level.

Salt-Freshwater Interface

► FIGURE 8.36
Circulation of fresh and saline ground water at a zone of diffusion in a coastal aquifer.
Source: H. H. Cooper, Jr., U.S. Geological Survey Circular 1613-C, 1964.



Depth to Salt/Fresh Water Interface

These early observers noted that in unconfined coastal aquifers, the depth to which fresh water extends below sea level is approximately 40 times the height of the water table above sea level. The (misnamed) **Ghyben-Herzberg principle** states that

$$z_{(x,y)} = \frac{\rho_w}{\rho_s - \rho_w} h_{(x,y)} \qquad (8.1)$$

where

$z_{(x,y)}$ is the depth to the salt-water interface below sea level at location (x,y) (L; ft or m)

$h_{(x,y)}$ is the elevation of the water table above sea level at point (x,y) (L; ft or m)

ρ_w is the density of fresh water (M/L^3; g/cm^3)

ρ_s is the density of salt water (M/L^3; g/cm^3)

The application of this principle is limited to situations in which both the fresh water and salt water are static.

Ghyben-Herzberg Ratio

EXAMPLE PROBLEM

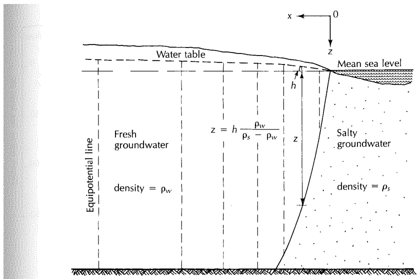
If $\rho_w = 1.000$ g/cm^3 and $\rho_s = 1.025$ g/cm^3, what is the ratio of $z_{(x,y)}$ to $h_{(x,y)}$?

$$z_{(x,y)} = \frac{\rho_w}{\rho_s - \rho_w} h_{(x,y)}$$

$$= \frac{1.000 \text{ g/cm}^3}{1.025 \text{ g/cm}^3 - 1.000 \text{ g/cm}^3} h_{(x,y)}$$

$$= 40 h_{(x,y)}$$

Ghyben-Herzberg



▲ FIGURE 8.37
Relationship of fresh-water head and depth to salt-water interface.

Flow in Coastal Aquifers

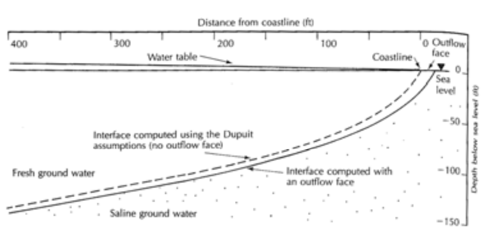
Flow in coastal aquifers can be described by means of the Dupuit equations in combination with the Ghyben-Herzberg principle. The steady flow of ground water is given by the partial differential equation (Fetter 1972b):

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = \frac{-2w}{K\,(1 + G)} \qquad \textbf{(8.2)}$$

where

w is the recharge to the aquifer (L/T; ft/day, m/day)
K is the hydraulic conductivity (L/T; ft/day, m/day)
G is equal to $\frac{\rho_w}{\rho_s - \rho_w}$ (dimensionless)

Flow in Coastal Aquifers



▲ FIGURE 8.38
Comparison of the position of the salt-water interface on eastern Long Island as computed using the Dupuit assumptions with that computed using an outflow face. *Source: C. W. Fetter, Jr.,* Water Resources *8 (1972): 1307–14. Used with permission.*

Flow in Coastal Aquifers

Should the value of the depth to the salt-water interface, as computed by Equation 8.1, exceed the depth of the aquifer, then the salt-water wedge is missing. This is the case on the left side of Figure 8.37. In this region, the governing equation is (Polubarinova-Kochina 1962)

$$\frac{\partial^2 h^2}{\partial x^2} + \frac{\partial^2 h^2}{\partial y^2} = \frac{-w}{K\,(z_m + h)} \qquad (8.3)$$

where z_m is the aquifer thickness below sea level. Both Equations 8.2 and 8.3 can be solved for an infinite-strip coastline, that is, one with flow in only one direction. The x- and z-axes are shown on Figure 8.37.

The *Dupuit-Ghyben-Herzberg model* of one-dimensional flow in coastal aquifers yields the following expression for the x- and z-coordinates of the interface (Todd 1953):

$$z = \sqrt{\frac{2q'xG}{K}} \qquad (8.4)$$

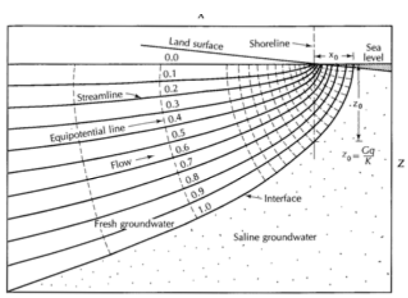
where q' is the discharge from the aquifer at the coastline, per unit width $[(L^3/T)/L]$.

Outflow Face

One of the failings of the Dupuit-Ghyben-Herzberg model of coastal aquifers is that the salt-water interface intercepts the water table at the coastline. This does not allow for vertical components of flow and discharge of the fresh water into the sea floor. Therefore, a simple model has been developed in which the x- and z-coordinates of the interface are given by the following relation (Glover 1964):

$$z = \sqrt{\frac{G^2 q'^2}{K^2} + \frac{2Gq'x}{K}} \qquad (8.5)$$

Outflow Face



▲ FIGURE 8.39
Flow pattern near a beach as computed using Equation 8.5. *Source: R. E. Glover, U.S. Geological Survey Water-Supply Paper 1613-C, 1964.*

Outflow Face

Note that Equation 8.5 is identical to 8.4, except that a constant, Gq'/K, has been added. Thus, when $x = 0$, z will still have a value. The interface is shown in Figure 8.39.

The width of the outflow face, x_0, may be found from the expression

$$x_0 = -\frac{Gq'}{2K} \tag{8.6}$$

and the height of the water table at any distance, x, from the coast is given by

$$h = \sqrt{\frac{2q'x}{GK}} \tag{8.7}$$

Islands

For an infinite-strip island receiving recharge at a rate, w, with a width equal to $2a$, the head of the water table, h, at any distance, x, from the shoreline is given by (Fetter 1972b):

$$h^2 = \frac{w[a^2 - (a - x)^2]}{K(1 + G)} \tag{8.8}$$

A circular island with a radius of distance R can be evaluated using

$$h^2 = \frac{w(R^2 - r^2)}{2K(1 + G)} \tag{8.9}$$

where h is the head above sea level at some radial distance, r, from the center of the island.

Island Example

Part A: An infinite-strip island has a width of 2 km. The permeability of the sediments is 10^{-2} cm/s and there is a daily accretion of 0.13 cm/day. The density of fresh water is 1.000 and the density of salty ground water is 1.025. Compute a water-table profile across the island using Equation 8.8. Then determine the interface depth using Equation 8.1.

$$G = \frac{1}{1.025 - 1} = 40$$

$$K = 10^{-2} \text{ cm/s} = 8.64 \text{ m/day}$$

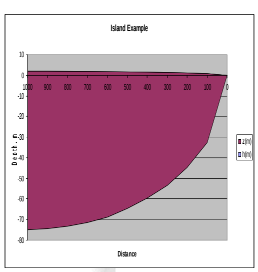
$$w = 0.0013 \text{ m/day}$$

$$a = 1000 \text{ m}$$

$$h^2 = \frac{w[a^2 - (a - x)^2]}{K(1 + G)}$$

$$z = Gh$$

x (m)	*h* (m)	*z* (m)
1000	1.92	76.8
900	1.91	76.4
800	1.88	75.2
700	1.83	73.2
600	1.76	70.4
500	1.66	66.4
400	1.53	61.2
300	1.37	54.8
200	1.15	46.0
100	0.84	33.6
0	0	0

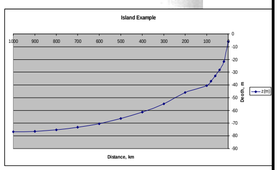


Coastline Example

Part B: From the computed profile, it can be seen that the fresh-water lens thins very rapidly in the last 100 m as the shoreline is approached. As the Dupuit assumptions may not be valid near the coastline, the profile of the interface close to the coast can be computed by use of Equation 8.5. The outflow per unit width, q', is equal to the recharge rate times the half-width.

$$q' = 0.0013 \text{ m}^3/\text{day} \times 1000 \text{ m}$$
$$= 1.3 \text{ m}^3/\text{day/m}$$
$$z = \frac{Gq'}{K} + \sqrt{\frac{2Gq'x}{K}}$$

x (m)	z (m)
0	6.0
20	21.5
40	28.0
60	32.9
80	37.1
100	40.7



Coastline Example

Find the width of the outflow face:

$$x_0 = -\frac{Gq'}{2K}$$
$$= -3.0 \text{ m}$$

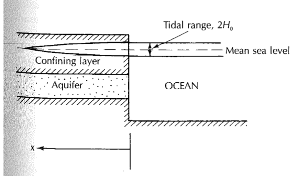
Find the height of the water table at a distance from the coast of 100 m:

$$h = \sqrt{\frac{2q'x}{GK}}$$
$$= 0.87 \text{ m}$$

Tidal Effects

The amplitude of the tidal change is H_0 and the tidal period, or time for the tide to go from one extreme to the other, is t_0. At any distance, x, inland from the coast, the amplitude of the tidal fluctuation, H_x, is given by (Jacob 1950)

$$H_x = H_0 \exp\left(-x\sqrt{\pi S/t_0 T}\right) \quad \textbf{(8.10)}$$



◀ FIGURE 8.40
Coastal aquifer showing the tidal range, $2H_0$, and the effect of the tide on the potentiometric surface.